Electromagnetic Fields

&

Electrical Machines

For

Electrical Engineering

By

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Syllabus for Electromagnetic Fields & Electrical Machines

Electromagnetic Fields: Coulomb's Law, Electric Field Intensity, Electric Flux Density, Gauss's Law, Divergence, Electric Field and Potential due to Point, Line, Plane and Spherical Charge Distributions, Effect of Dielectric Medium, Capacitance of Simple Configurations, Biot-Savart's Law, Ampere's Law, Curl, Faraday's Law, Lorentz Force, Inductance, Magnetomotive Force, Reluctance, Magnetic Circuits, Self and Mutual Inductance of Simple Configurations.

Electrical Machines: Single Phase Transformer: Equivalent Circuit, Phasor Diagram, Open Circuit and Short Circuit Tests, Regulation and Efficiency, Three Phase Transformers, Connections, Parallel Operation, Auto-Transformer, Electromechanical Energy Conversion Principles, DC Machines, Separately Excited, Series and Shunt, Motoring and Generating Mode of Operation and Their Characteristics, Starting and Speed Control of DC Motors, Three Phase Induction Motors, Principle of Operation, Types, Performance, torque-Speed Characteristics, No-Load and Blocked Rotor Tests, Equivalent Circuit, Starting and Speed Control, Operating Principle of Single Phase Induction Motors, Synchronous Machines: Cylindrical and Salient Pole Machines, Performance, Regulation and Parallel Operation of Generators, Starting of Synchronous Motor, Characteristics, Types of Losses and Efficiency Calculations of Electric Machines.

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"Picture yourself vividly as winning and that alone will contribute immeasurably to success."

...Harry Fosdick

CHAPTER 1

Electromagnetic Field

Learning Objectives

After reading this chapter, you will know:

1. Elements of Vector Calculus
2. Operators, Curl, Divergence
3. Electromagnetic Coulombs' law, Electric Field Intensity, Electric Dipole, Electric Flux Density
4. Gauss's Law, Electric Potential
5. Divergence of Current Density and Relaxation
6. Boundary Conditions
7. Biot-Savart's Law, Ampere Circuit Law, Continuity Equation
8. Magnetic Vector Potential, Energy Density of Electric & Magnetic Fields, Stored Energy in Inductance
9. Faraday's Law, Motional EMF, Induced EMF Approach
10. Maxwell's Equations

Introduction

Cartesian coordinates (x, y, z), $-\infty < x < \infty, -\infty < y < \infty, -\infty < z < \infty$

Cylindrical coordinates (ρ, ϕ, z), $0 \le \rho < \infty, 0 \le \phi < 2\pi, -\infty < z < \infty$

Spherical coordinates (r, θ, ϕ), $0 \le r < \infty, 0 \le \theta \le \pi, 0 \le \phi < 2\pi$

Other valid alternative range of θ and ϕ are-----

(i) $0 \le \theta < 2\pi, 0 \le \phi \le \pi$

(ii) $-\pi \le \theta \le \pi, 0 \le \phi \le \pi$

(iii) $-\frac{\pi}{2} \le \theta \le {}^{\pi}/_{2}, 0 \le \phi < 2\pi$

(iv) $0 < \theta \le \pi, -\pi \le \phi < \pi$

Vector Calculus Formula

SL. No	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
(a)	Differential Displacement $dl = dx\, a_x + dy\, a_y + dz\, a_z$	$dl = d\rho a_\rho + \rho d\phi a_\phi + dz a_z$	$dl = dr a_r + r d\theta a_\theta + r \sin\theta d\phi a_\phi$
(b)	Differential Area $dS = dy\, dz\, a_x$ $= dx\, dz\, a_y$ $= dx\, dy\, a_z$	$dS = \rho\, d\phi\, dz\, a_\rho$ $= d\rho\, dz\, a_\phi$ $= \rho d\rho d\phi\ a_z$	$d_s = r^2 \sin\theta\, d\theta\, d\phi\, a_r$ $= r \sin\theta\, dr\, d\phi\, a_\theta$ $= r\, dr\, d\theta\, a_\phi$
(c)	Differential Volume $dv = dx\, dy\, dz$	$dv = \rho\, d\rho\, d\phi\, dz$	$dv = r^2 \sin\theta\, d\theta\, d\phi\, dr$

Operators

1) ∇ V – Gradient, of a Scalar V
2) $\nabla \overline{V}$ – Divergence, of a Vector $\overline{V}$
3) $\nabla \times \overline{V}$ – Curl, of a Vector $\overline{V}$
4) ∇^2 V – Laplacian, of a Scalar V

DEL Operator:

$$\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z \text{(Cartesian)}$$
$$= \frac{\partial}{\partial \rho} a_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi} a_\phi + \frac{\partial}{\partial z} a_z \text{(Cylindrical)}$$
$$= \frac{\partial}{\partial r} a_r + \frac{1}{r}\frac{\partial}{\partial \theta} a_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi} a_\phi \text{(Spherical)}$$

Gradient of a Scalar field

V is a vector that represents both the magnitude and the direction of maximum space rate of increase of V.

$$\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z \text{ For Cartisian Coordinates}$$
$$= \frac{\partial V}{\partial \rho} a_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi} a_\phi + \frac{\partial V}{\partial z} a_z \text{ For Spherical Coordinates}$$
$$= \frac{\partial V}{\partial r} a_r + \frac{1}{r}\frac{\partial V}{\partial \theta} a_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi} a_\phi \text{ For Cylindrical Coordinates}$$

The following are the fundamental properties of the gradient of a scalar field V

1. The magnitude of ∇V equals the maximum rate of change in V per unit distance.
2. ∇V points in the direction of the maximum rate of change in V.
3. ∇V at any point is perpendicular to the constant V surface that passes through that point.
4. If A = ∇V, V is said to be the scalar potential of A.
5. The projection of ∇V in the direction of a unit vector a is ∇V. a and is called the directional derivative of V along a. This is the rate of change of V in direction of a.

Example: Find the Gradient of the following scalar fields:

(a) $V = e^{-z} \sin 2x \cosh y$
(b) $U = \rho^2 z \cos 2\phi$
(c) $W = 10r \sin^2\theta \cos\phi$

Solution:

(a) $\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$
$= 2e^{-z} \cos 2x \cosh y\, a_x + e^{-z} \sin 2x \sinh y\, a_y - e^{-z} \sin 2x \cosh y\, a_z$

(b) $\nabla U = \frac{\partial U}{\partial \rho} a_\rho + \frac{1}{\rho}\frac{\partial U}{\partial \phi} a_\phi + \frac{\partial U}{\partial z} a_z$
$= 2\rho z \cos 2\phi\, a_\rho - 2\rho z \sin 2\phi\, a_\phi + \rho^2 \cos 2\phi\, a_z$

(c) $\nabla W = \frac{\partial W}{\partial r} a_r + \frac{1}{r}\frac{\partial W}{\partial \theta} a_\theta + \frac{1}{r\sin\theta}\frac{\partial W}{\partial \phi} a_\phi$
$= 10 \sin^2\theta \cos\phi\, a_r + 10 \sin 2\theta \cos\phi\, a_\theta - 10 \sin\theta\, \sin\phi\, a\phi$

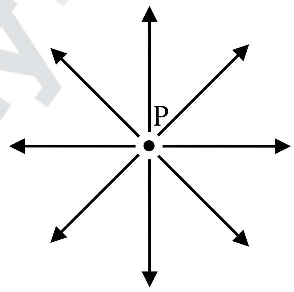
Divergence of a Vector

Statement: Divergence of $\overline{A}$ at a given point P is the outward flux per unit volume as the volume shrinks about P.

Hence,

$$\text{Div}A = \nabla . A = \lim_{\Delta v \to 0} \frac{\oint_S A . ds}{\Delta v} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where, Δv is the volume enclosed by the closed surface S in which P is located. Physically, we may regard the divergence of the vector field A at a given point as a measure of how much the field diverges or emanates from that point.



$$\nabla . A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y}\frac{\partial A_z}{\partial z} \quad \text{Cartisian System}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \quad \text{Cylindrical System}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi} \quad \text{Sphearical System}$$

From equation (1),

$$\oint_S A . dS = \int_V \nabla . A \, dv$$

This is called divergence theorem which states that the total outward flux of the vector field A through a closed surface S is same as the volume integral of the divergence of A.

Example: Determine the divergence of these vector field

(a) $P = x^2yza_x + xza_z$

(b) $Q = \rho \sin\phi \; a_\rho + \rho^2 z a_\phi + z\cos\phi \; a_z$

(c) $T = \frac{1}{r^2}\cos\theta \; a_r + r\sin\theta\cos\phi \; a_\theta + \cos\theta \, a_\phi$

Solution:

(a) $\nabla . P = \frac{\partial}{\partial x}P_x + \frac{\partial}{\partial y}P_y + \frac{\partial}{\partial z}P_z$

$$= \frac{\partial}{\partial x}(x^2yz) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(xz)$$

$$= 2xyz + x$$

(b) $\nabla . Q = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho Q_\rho) + \frac{1}{\rho}\frac{\partial}{\partial \phi}Q_\phi + \frac{\partial}{\partial z}Q_z$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho^2 \sin\phi) + \frac{1}{\rho}\frac{\partial}{\partial \phi}(\rho^2 z) + \frac{\partial}{\partial z}(z\cos\phi)$$

$$= 2\sin\phi + \cos\phi$$

(c) $\nabla . T = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 T_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(T_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(T_\phi)$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(\cos\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(r\sin^2\theta\cos\phi) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\cos\theta)$$

$$= 0 + \frac{1}{r\sin\theta}2r\sin\theta\cos\theta\cos\phi + 0$$

$$= 2\cos\theta\cos\phi$$

Curl of a Vector

Curl of a Vector field provides the maximum value of the circulation of the field per unit area and indicates the direction along which this maximum value occurs.

That is,

$$\text{Curl } A = \nabla \times A = \lim_{\Delta S \to 0} \left(\frac{\oint_L A \, . \, dl}{\Delta S} \right)_{max} a_n \dots\dots\dots\dots\dots\dots\dots . . (2)$$

$$\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \frac{1}{\rho} \begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \dfrac{\partial}{\partial \rho} & \dfrac{\partial}{\partial \phi} & \dfrac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$= \frac{1}{r^2 \sin\theta} \begin{vmatrix} a_\rho & r a_\theta & r \sin\theta \, a_\phi \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial \theta} & \dfrac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin\theta \, A_\phi \end{vmatrix}$$

From equation (2) we may expect that

$$\oint_L \mathbf{A \, dl} = \int_S (\nabla \times \mathbf{A}) \, . \, \mathbf{ds}$$

This is called stoke's theorem, which states that the circulation of a vector field A around a (closed) path L is equal to the surface integral of the curl of A over the open surface S bounded by L, Provided A and $\mathbf{\Delta} \times \mathbf{A}$ are continuous no s.

Example: Determine the curl of each of the vector fields.

(a) $\mathbf{P = x^2 yz \, a_x + xza_z}$

(b) $\mathbf{Q = \rho \sin\phi a_\rho + \rho^2 \, za_\phi + z \cos \phi a_z}$

(c) $\mathbf{T = \frac{1}{r^2} \cos\theta \;\; a_r + r \sin\theta \cos\phi a_\theta + \cos\phi \; a_\phi}$

Solution:

(a) $$\nabla \times \mathbf{P} = \left(\frac{\partial P_z}{\partial y} - \frac{\partial P_y}{\partial z} \right) a_x + \left(\frac{\partial P_x}{\partial z} - \frac{\partial P_z}{\partial x} \right) a_y + \left(\frac{\partial P_y}{\partial x} - \frac{\partial P_x}{\partial y} \right) a_z$$

$$= (0 - 0) a_x + (x^2 y - z) a_y + (0 - x^2 z) a_z$$

$$= (x^2 y - z) a_y - x^2 z a_z$$

(b) $$\nabla \times \mathbf{Q} = \left[\frac{1}{\rho} \frac{\partial Q_z}{\partial \phi} - \frac{\partial Q_\phi}{\partial z} \right] a_\rho + \left[\frac{\partial Q_\rho}{\partial z} - \frac{\partial Q_z}{\partial \rho} \right] a_\phi + \frac{1}{\rho} \left[\frac{\partial}{\partial \rho} (\rho Q_\phi) - \frac{\partial Q_\rho}{\partial \phi} \right] a_z$$

$$= \left(\frac{-z}{\rho} \sin\phi - \rho^2 \right) a_\rho + (0 - 0) a_\phi + \frac{1}{\rho} (3\rho^2 z - \rho \cos\phi) a_z$$

$$= -\frac{1}{\rho} (z \sin\phi + \rho^3) a_\rho + (3\rho z - \cos\phi) a_z$$

$$\text{(c)}\quad \nabla\times T = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(T_\phi\sin\theta) - \frac{\partial}{\partial\phi}T_\theta\right]a_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}T_r - \frac{\partial}{\partial r}(rT_\phi)\right]a_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(rT_\theta) - \frac{\partial}{\partial\theta}T_r\right]a_\phi$$

$$= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\cos\theta\sin\theta) - \frac{\partial}{\partial\phi}(r\sin\theta\cos\phi)\right]a_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}\frac{(\cos\theta)}{r^2} - \frac{\partial}{\partial r}(r\cos\theta)\right]a_\theta + \frac{1}{r}\left[\frac{\partial}{\partial r}(r^2\sin\theta\cos\phi) - \frac{\partial}{\partial\theta}\frac{(\cos\theta)}{r^2}\right]a_\phi$$

$$= \frac{1}{r\sin\theta}(\cos 2\theta + r\sin\theta\sin\phi)a_r + \frac{1}{r}(0-\cos\theta)a_\theta + \frac{1}{r}\left(2r\sin\theta\cos\phi + \frac{\sin\theta}{r^2}\right)a_\phi$$

$$= \left(\frac{\cos 2\theta}{r\sin\theta} + \sin\phi\right)a_r - \frac{\cos\theta}{r}a_\theta + \left(2\cos\phi + \frac{1}{r^3}\right)\sin\theta\ a_\phi$$

Laplacian

(a) Laplacian of a scalar field V, is the divergence of the gradient of V and is written as $\nabla^2 V$.

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \rightarrow \textbf{For Cartisian Coordinates}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2} \rightarrow \textbf{For Cylindrical Coordinates}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin\theta}\frac{\partial^2 V}{\partial\phi^2} \rightarrow \textbf{For Spherical Coordinates}$$

If $\nabla^2 V = 0$, V is said to be harmonic in the region.

A vector field is solenoid if $\nabla . A = 0$; it is irrotational or conservative if $\nabla\times A = 0$

$\nabla.(\nabla \times A) = 0$

$\nabla\times(\nabla V) = 0$

(b) Laplacian of Vector $\overline{A}$

$\nabla^2\vec{A} = \cdots$ is always a vector quantity

$\nabla^2\vec{A} = (\nabla^2 A_x)\hat{a}x + (\nabla^2 A_y)\hat{a}y + (\nabla^2 A_z)\hat{a}z$

$\nabla^2 A_x \rightarrow$ Scalar quantity

$\nabla^2 A_y \rightarrow$ Scalar quantity

$\nabla^2 A_z \rightarrow$ Scalar quantity

$\nabla^2 V = \frac{-p}{\epsilon}$........Poission's E.q.

$\nabla^2 V = 0$ ....Laplace E.q.

$\nabla^2 E = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu E\frac{\partial^2 E}{\partial t^2}$....... wave E. q.

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"Obstacles are those frightful things you can see when you take your eyes off your goal."

...Henry Ford

CHAPTER 1

Transformers

Learning Objectives

After reading this chapter, you will know:

1. Constructional Details of Transformers
2. Equivalent Circuit of Transformers
3. Voltage Regulation of a Transformers
4. Testing of Transformer
5. Parallel Operation of Transformers
6. 3-Phase Transformation
7. Auto-Transformers

Introduction

A transformer basically is very simple static (or stationary) electro-magnetic passive electrical device that works on the principle of Faraday's law of induction by converting electrical energy from one value to another.

The transformer does this by linking together two or more electrical circuits using a common oscillating magnetic circuit which is produced by the transformer itself. A transformer operates on the principals of "electromagnetic induction", in the form of Mutual Induction.

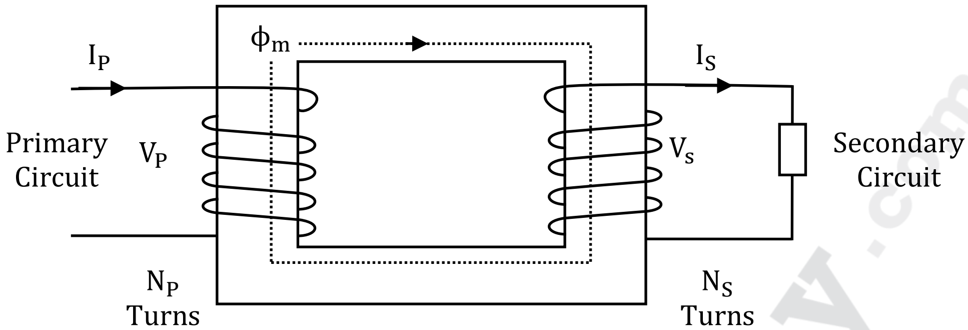
Mutual induction is the process by which a coil of wire magnetically induces a voltage into another coil located in close proximity to it. Then we can say that transformers work in the "magnetic domain", and transformers get their name from the fact that they "transform" one voltage or current level into another.

Transformers are capable of either increasing or decreasing the voltage and current levels of their supply, without modifying its frequency, or the amount of Electrical Power being transferred from one winding to another via the magnetic circuit.

A single phase voltage transformer basically consists of two electrical coils of wire, one called the "Primary Winding" and another called the "Secondary Winding". We will define the "primary" side of the transformer as the side that usually takes power, and the "secondary" as the side that usually delivers power. In a single-phase voltage transformer the primary is usually the side with the higher voltage.

These two coils are not in electrical contact with each other but are instead wrapped together around a common closed magnetic iron circuit called the "core". This soft iron core is not solid but made up of individual laminations connected together to help reduce the core's losses.

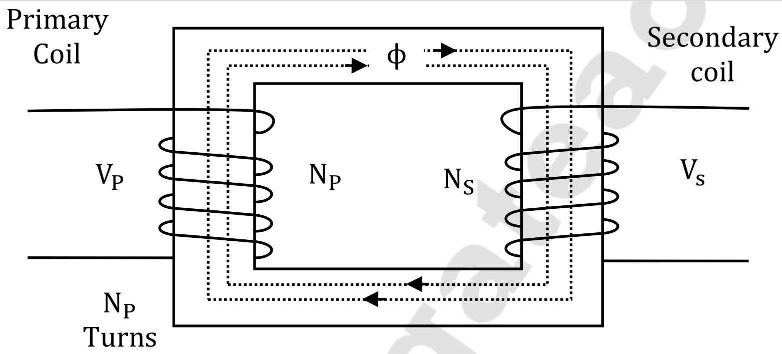
The two coil windings are electrically isolated from each other but are magnetically linked through the common core allowing electrical power to be transferred from one coil to the other. When an electric current passed through the primary winding, a magnetic field is developed which induces a voltage into the secondary winding as shown.

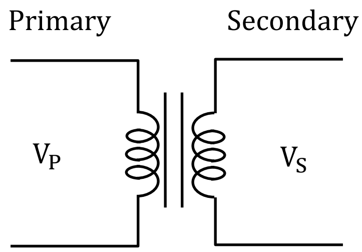


Single Phase Voltage Transformer

In other words, for a transformer there is no direct electrical connection between the two coil windings, thereby giving it the name also of an Isolation Transformer. Generally, the primary winding of a transformer is connected to the input voltage supply and converts or transforms the electrical power into a magnetic field. While the job of the secondary winding is to convert this alternating magnetic field into electrical power producing the required output voltage as shown.

Constructional Details of Transformers





Where,
V_P = Primary Voltage
V_S = Secondary Voltage
N_P = Number of Primary Windings
N_S = Number of Secondary Windings
ϕ = Flux Linkage

Core:

- The core provides a path of low reluctance.
- The relative permeability for the core material is of the order of 1,000
- Silicon steel or sheet steel with 4% silicon is used.
- The core plates of a transformer are made of silicon steel or sheet steel.
- The sheets are laminated and stacked to reduce eddy current losses.
- The sheets are laminated and coated with an oxide to reduce iron losses.

- The thickness of lamination is 0.35 mm for 60 Hz operation.
- The thickness of lamination is 5 mm for 25 Hz operation.
- For a given value of flux, the primary Ampere Turn required are less if the reluctance is low.
- A spiral core is assembled using continuous strip of transformer silicon steel wound in the form of a circular or elliptical cylinder.
- In a spiral core transformer higher flux densities can be used.
- A spiral core transformer has lower loss per Kg. Weight.

Windings
- Conventional transformer has two windings.
- The winding which receives electrical energy is called primary winding.
- The winding which delivers electrical energy is called secondary winding.
- Windings are made of High grade copper.
- Stranded conductors are used for windings carrying higher currents to reduce eddy current loss.
- Additional insulation is provided for line end connections, because during disturbances (switching over voltages and lightning) 80% of the voltage appears across the first 10% of turns from the line end.
- For large power and distribution transformers, an oil-filled tank is necessary for cooling the windings and the core.
- Two types of losses: Core and copper, occur during operation.
- Heat produced is roughly proportional to the volume of the material in which losses occur.
- Heat dissipation is proportional to the surface area of the same material and the tank.
- The surface is corrugated to increase the surface area. Radiators are also used.

Methods of Cooling
(a) Natural Radiation------low voltage and output ratings. (500V, 5 kVA)
(b) Oil filled and self-cooled------large sized transformers. (132 kV, 100 MVA)
(c) Forced cooling with air blast------Transformers with ratings higher than 33 kV and 100 MVA

Conservator Tank
- Due to variations in load and climatic conditions, the oil in oil-filled, self-cooled transformers expands or contracts.
- In the absence of a conservator tank, high pressures are developed which may burst the tank.

Bushings
- To provide proper insulation to the output leads to be taken from the transformer tank.
- Porcelain type bushings are used up to 33 kV.
- Condenser type and oil-filled type bushings are used beyond 33 kV.

Breather
- Absorption of moisture and dust by oil must be prevented.
- To prevent moisture and dust from entering the conservator tank oil, breather is provided.

Types of Transformers

- Core type: Copper windings surround core.
- Shell type: Iron core surrounds the copper windings.
- To reduce the eddy currents induced in the core, thin laminations are used.
- To reduce the hysteresis loss, heat treated grain oriented COLD ROLLED GRAIN ORIENTED i.e., (CRGO) silicon steel laminations are used.

Core-Type Transformers

There are two types of core-type transformers, they are

1. Core-type
2. Distributed core type

- In a simple core-type transformer, there is a single magnetic circuit.
- The vertical members of the core are called limbs, and the horizontal members are called yokes.
- Each limb of a core-type transformer carries a half of primary windings and a half of secondary windings.
- In a distributed-core type transformer, the windings are on the central limb.
- The number of parallel magnetic circuits in a distributed core type transformer is equal to the number of parts of distributed core.
- Because of the presence of insulating materials, the core area gets reduced by about 10%
- Iron factor is the ratio of active area of core and gross area of core and its value is approximately 0.9
- Used for high voltage applications.

Shell-Type Transformers

- A shell type transformer has two magnetic circuits parallel to each other.
- To reduce the mechanical vibrations and humming noise, the transformers are provided with good bracing.
- Humming noise is due to MAGNETOSTRICTION, of the core due to varying flux.
- Used for low voltage applications.

Principle of Operation

- Transformer works on the principle of mutual induction.
- The voltage per turn of the primary and secondary windings is the same since the same mutual flux cuts both the windings, if both the windings are identical in cross-section.
- The ratio of the induced EMF'S = Ratio of the turns.
- Since $E_1 \cong V_1$ and $E_2 \cong V_2$ $\therefore V_1/V_2 = T_1/T_2$
- In a loaded transformer, the primary draws a current so that mutual flux is maintained constant.
- Since no-load primary Ampere turns are very small compared to full-load Ampere turns, $I_1T_1 = I_2T_2$
- $I_1/I_2 = T_2/T_1 = V_2/V_1$ i.e., $V_1I_1 = V_2I_2$ Primary VA = Secondary VA

E.M.F Equation

Voltage applied to the primary and the magnetic flux set up in the core are assumed to be sinusoidal.

If $\phi = \phi_m \sin \omega t$ $(\omega = 2\pi f)$

$e_1 = -T_1 (d\phi/dt) = -T_1 \omega \phi_m \cos \omega t = T_1 \omega \phi_m \sin(\omega t - 90°)$

$e_2 = -T_2 (d\phi/dt) = -T_2 \omega \phi_m \cos \omega t = T_2 \omega \phi_m \sin(\omega t - 90°)$

Similarly,

$E_{1max} = -T_1 \omega \phi_m = 2\pi f T_1 \phi_m$

$E_1 = \left(\frac{1}{\sqrt{2}}\right) E_{1max} = \frac{2\pi f}{\sqrt{2}} T_1 \phi_m = 4.44 f \times T_1 \times \phi_m$

Similarly,

$E_2 = 4.44 f \times T_2 \times \phi_m$ volts/phase

E_1 and E_2 are in phase and lag behind ϕ_m by an angle of 90°

Losses and Efficiency

- Since a transformer is a static device, there are no mechanical losses.
- There will be only magnetic (hysteresis and eddy current losses) and copper losses due to the flow of current through the windings.
- Hysteresis loss is proportional to the maximum value of flux density raised to the power of 1.6 and the supply frequency i.e., $B_m^{1.6} f$
- The eddy current losses are proportional to the square of the maximum flux density and the square of the frequency and the square of thickness of laminations. i.e., $B_m^2 f^2 t^2$
- The flow of current through the windings gives rise to the copper losses, viz., $I_1^2 r_1$ and $I_2^2 r_2$
- The magnetic losses are present as long as the primary is energized.
- Since the no load current is only of the order of 5% of the rated or full load current, the no load copper loss in the primary winding is neglected. So, the no load input to a transformer is taken as the magnetic loss or the iron or the core loss. It is assumed to be same under all operating conditions, right from no load to full load (or even slight over load). It is denoted as P_i
- The copper losses vary with the value of the secondary (and hence the Primary) current. The copper loss corresponding to the rated value of the current is called the full load copper loss. We shall designate it as P_c
- The efficiency (sometimes called the commercial efficiency) of a transformer is the ratio of the power output and power input, both expressed in the same units (Watts, Kilowatts or Megawatts).
- Let kVA be the rated of the transformer, x be the fraction of the full load at which the transformer is working ($0 \le x \le 1.0$ usually), and $\cos \phi$ be the power factor of the load. Then the efficiency is given by

  $$\eta = \frac{x \times kVA \times \cos \phi}{x \times kVA \times \cos \phi + x^2 P_c + P_i}$$

  Where P_c is the I^2R loss at full load
- For a given load, the power factor is constant. So, by differentiating 'η' with respect to 'x' and setting it to zero, we obtain the condition for the maximum efficiency as variable copper losses = constant iron losses, i.e., $x^2 P_c = P_i$
- At maximum efficiency operation, the total losses = $2 P_i = 2x^2 P_c$, since $x = \sqrt{(P_i/P_c)}$